**THERMAL EQUILIBRIUM BETWEEN TWO BODIES**

**Objective:**

The activity aims at making the student realize that:

- thermal equilibrium is not reached instantaneously;
- investigating thermal equilibrium between two bodies is easier when they are thermally insulated from the environment;
- a model of the phenomenon can be constructed using exponential function.

Pierre Simon Laplace (1749-1827), French mathematician and astronomer, is one of the most important scientists of the Napoleonic period; its research notably contributed to various fields of Mathematics, but also Astronomy, Thermodynamics and Statistics. He was D’Alembert’s scholar and started his career as teacher at the King’s Military School. Laplace built one of the first ice calorimeters. His work followed that of Johann Karl Wilcke (1732-1796) who defined the calorie and introduced the concept of specific heat. (partly taken from Wikipedia, the free encyclopaedia)

The experience consists in following the processes of heating and cooling of two objects that exchange heat in an adiabatic container. The two temperature are simultaneously measured as functions of time.
THEORY

When a body of mass $m_1$, at temperature $T_1$, is put in thermal contact with another body of mass $m_2$, at temperature $T_2$ different from $T_1$, heat transfer occurs from the body at higher temperature to the body at lower temperature. The heat flux ends as soon as the two bodies reach the same temperature $T_e$, i.e. when they have reached the thermal equilibrium.

If the two bodies are kept in an adiabatic container there will be no loss of heat to the surroundings, therefore all the heat lost by the warmer body will be gained by the cooler one.

If $Q_1$ is the heat lost by the first body and $c_1$ is the specific heat of the stuff the body is made of, we have

$$Q_1 = m_1 c_1 (T_1 - T_e) \tag{1}$$

If $Q_2$ is the heat gained by the second body and $c_2$ the corresponding specific heat, we have

$$Q_2 = m_2 c_2 (T_e - T_2) \tag{2}$$

By equating the quantities $Q_1$ and $Q_2$ and solving for the equilibrium temperature $T_e$, we have:

$$T_e = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2} \tag{3}$$

The simpler case is when the two bodies are identical, i.e. are made of the same stuff (same specific heat) and have the same mass. In this case the equilibrium temperature is the mean of the initial temperatures of the two bodies.

It is interesting to investigate the process. The cooling and the heating processes are described by exponential functions.

If $\Delta T$ is the difference between the instantaneous temperature of the body and the equilibrium temperature and $\Delta T_0$ is the same difference at time zero, we get:

$$\Delta T = \Delta T_0 e^{-ht} \tag{4}$$

The parameter $h$ depends upon the geometry of the system and gets higher as thermal exchange becomes quicker.

The value of $h$ can be determined by transforming the exponential relationship into a linear one. Using logarithm properties we get:

$$\ln(T - T_e) = \ln(T_0 - T_e) - ht \tag{5}$$
EXPERIMENTAL SETUP

This experiment has been tried with different setups in order to find how to perform a simple experiment that could also give reliable data for constructing a mathematical model of the phenomenon.

In calorimetric measurements usually the process of reaching the equilibrium is not investigated. On-line instruments on the contrary allow the development of the temperatures in time to be recorded and facilitate understanding how equilibrium is reached and checking how much real phenomenon and mathematical model match each other.

We have tried to allow two liquids to exchange heat without mixing up by filling a plastic balloon with one of them. As adiabatic container we have used a thermos flask, as shown in the photo. This system has the advantage to be easily constructed also outside the laboratory, but it proved not to be insulating enough, therefore the data analysis becomes rather complex. Problems on arranging the probes may also occur and measures are not reliable if they touch the walls of the container or the surface of the balloon. (fig 1)

![Fig.1: thermos flask.](image)

We have also tried with metallic cylinders with holes for the probes to be inserted into. But this system has the same disadvantages of the first one and is less flexible because it does not allow for changes of the masses and the materials of the bodies.

![Fig. 2: metallic cylinders with holes for the probes to be inserted into.](image)
The best system we found up to now is made with styrofoam food containers. It is very flexible and allows for many variations of the experiment. We put some water, at slightly lower temperature than room temperature, directly into the container and the same quantity of hot water in a styrofoam glass used for hot drinks. Thermal exchange is very slow and the time for reaching equilibrium is very long, but this is not a problem if automatic data acquisition is used.

![Fig 3: styrofoam food containers.](image)

The experimental apparatus is made of:

- A styrofoam container
- A styrofoam top with holes for the probe cables
- One or more styrofoam glasses
- Some fresh water (some ten degrees less than room temperatre)
- Some hot water
- Some other liquids at different temperatures (optional)
- Graphic calculator
- CBL2 interface
- Two steel or flexible temperature probes
- Black connecting cable (see photo)
- DataMate program
- TI-GRAPH LINK cable and TI Connect software

Different calculators or acquisition systems can be used.
EXPERIMENTAL SETUP

- Setup the acquisition system
- Let the temperature probes through the holes in the top and arrange them for measuring the temperature of the water in the glass and that of the water in the container
- Pour the hot water in the glass and the cold water in the container
- Start data acquisition (if you need help go to the Data Acquisition page)

We have collected 120 samples every 60 seconds. The measurement lasted 2 hours. Repeat the experiment changing the type of liquid or the masses.

How have your plots come out?

You can compare them with those we obtained (using two different kind of glasses – Styrofoam, thin plastic). If you cannot perform your own experiment, you can use the data we collected (go to Data Sample at the bottom of this page).

The teacher can find some suggestions and the graphs in the section about educational issues.
DATA ACQUISITION (TI 84 Plus)

In this experiment we shall collect the temperatures measured by the three probes. Measures can be collected using DataMate program.

Procedure can be divided into two stages: setup and collection.

Connect all cables:
- the calculator to the interface with the black cable (see photo)
- the probes to channels CH1, CH2 (see photo)

Arrange all necessary materials:
- fill the glass and the container with the same quantities of hot and cold water
- Start the DataMate program
- CHECKING SENSOR appears and near the channel labels CH 1: TEMP (C) - CH 2: TEMP (C) the temperature of each probe can be read.

To setup the measurement
- select 1:SETUP by pressing 1
- select MODE:TIME GRAPH and press ENTER
- from the menu SELECT MODE select 2:TIMEGRAPH
- select 2:CHANGE TIME SETTINGS
- when ENTER TIME BETWEEN SAMPLE appears press 60
- when ENTER NUMBER OF SAMPLES appears press 120, then ENTER
- to confirm select 1:OK by pressing 1

Insert the probes in the containers through the holes in the top and start measurements by selecting 2:START.

While measurement occurs the plots of the temperatures from the two probes is shown in real time; when the collection of data is over the plot is rescaled.

Collected data are recorded in the lists
- L1 time
- L2, L3 temperature

To have a look at the plot obtained on web site click on Styrofoam glass, plastic glass, or go to the Excel document in Data analysis (MEExcel).

It may be noticed that in the first case two hours are not enough to reach the equilibrium.

It is advisable to set the system so that thermal equilibrium occurs near to room temperature in order to make heat exchange with the surroundings as slow as possible. For this purpose we suggest to use hot water and water kept in the fridge for a few hours. Before starting to collect data it is better to wait a few minutes to let the system become stable.
DATA SAMPLE TI84

Data have been collected using:
- TI84 graphing calculator
- CBL2 interface (see photo)
- Two flexible temperature probes
- Black connecting cable (see photo)
- DataMate program
- TI-GRA PH LINK TM cable and software (optional)
- PC with TI Connect software (optional)


Original data (TI84) can be found on web site in:

Styrofoam glass:
- \( L_1 \) times
- \( L_2 \) first probe temperature
- \( L_3 \) second probe temperature

Plastic glass:
- \( L_1 \) times
- \( L_2 \) first probe temperature
- \( L_3 \) second probe temperature

Sample data (MS Excel):

In order to see or download sample data in MS Excel:
- click here, for the first set of measurements
- click here, for the second set of measurements.
DATA ANALYSIS (TI 84)

Data reduction with graphing calculator is pretty complex because selection of data, differences of temperatures and logarithm calculation. Data transfer to PC and the use of a specific software for data analysis is suggested (see Data Analysis Ms Excel at the bottom of this page).

DATA ANALYSIS (MS Excel™)

Plots can be visualized and analysed using a specific program or a MS Excel™ sheet.

Transferring experimental data to the PC

When collection is finished data can be transferred from the calculator to the PC. For this purpose you need a TI GRAPH LINK cable (see photo on web site) and TI Connect Software that allows the content of the calculator to be explored (TI DEVICE EXPLORER) and data to be edited (TI DATA EDITOR) that allows the content of the calculator to be explored (TI DEVICE EXPLORER) and data to be edited (TI DATA EDITOR).

Data are recorded in the calculator as follows:
- time in L1
- temperatures in L2, L3, L4

Instruction for data transfer from calculator to PC are available on web site.
Within TI Connect™, using the TI DEVICE EXPLORER option, lists can be saved in the PC. The list content can be visualized using TI DATA EDITOR and pasted in a MS Excel™ sheet. Sample data in MS Excel format are available for the two examples: example 1, example 2.

The system does not reach the equilibrium in two hours. Temperatures differ by more than 1°C. The mean between the two final temperatures can be taken as the final value at the equilibrium (Te in formula).

In other two columns ΔT for the two sets of temperature can be calculate as:
ΔT_1 = T_1 - T_e
and for the body at lower temperature that warms up as:
ΔT_2 = T_e - T_2
In two more columns natural logarithm of ΔT_1 and ΔT_2 can be calculated.

Plotting lnΔT_1 and lnΔT_2 as a function of time a linear relationship, at least in the middle section of data, is obtained. Excel allows a trend line to be drawn across the data and gives the values of the parameters of the fitting line.
In the sample data in the Excel sheet plots and data analysis both on the whole set of data and on the selection of the middle section are shown.
When heat is exchanged too quickly with the surroundings a thermal drift can be noticed as shown in the figure.

When analysing the data it is important to be careful in estimating the equilibrium temperature and in selecting the interval where to apply the mathematical model.
TEACHER’S GUIDE

The time necessary for this measurement can be very long according to the system considered. It is advisable to start the experiment and to have the class working on another activity meanwhile.

The class can be divided into groups and a different system can be given to each of them. It is useful to ask the students to prepare the adiabatic system by themselves using materials from everyday life. Various containers can be used (thermos, boxes of Styrofoam or other materials and of different shapes) that are likely to be more or less adiabatic; liquids can be changed (water, oil, alcohol, etc.); the smaller container can be changed (a plastic balloon, insulating or not insulating glasses); the masses of the liquids can also be changed. Each group should take care of measuring and recording the masses of liquids they use that must be chosen also according to the type of system.

Also a very long measurement (12-16 hours) of the temperature of a glass of hot water can be set up. It is surprising how long it takes for the water to reach thermal equilibrium with the surroundings.

By setting different activities the students are encouraged to look for analogies and differences between one situation and the other. When data collection and analysis are completed, each group illustrates and share with the others the results obtained; the teacher can stimulate the discussion and invite all students to compare the different cases analysed. Students get accustomed to discussion and collaboration, which are basic elements of scientific research.

The following concepts can be made clear and reinforced:
- Adiabatic system (it appears that a perfectly adiabatic system does not exist in reality, a thermal drift being always present)
- Thermal equilibrium (it appears that it is not instantaneous; systems may take a long time to reach the equilibrium; this is often neglected in theoretical teaching of Thermodynamics, where only quasi-static phenomena are considered)
- Specific heat capacity
- Heat capacity
- Heat transmission

This experiment provides a nice example of a phenomenon that continuously decreases gradually getting slower and slower. It is an opportunity for introducing exponential behaviours that are frequently found in phenomena from different fields of Physics (discharging a capacitor, emptying a tube full of water, …)

Data analysis can be performed differently according to the age of the students. For the younger ones the description of cooling and heating curves and the comparison of different cases will be enough; students more competent in Maths can develop the linear description of the phenomenon and calculate the parameters of the model.

The experiences are also described in:
G.Pezzi, Esperimenti on-line sul raffreddamento di un corpo, Ipotesi, anno 1, n°1, 1998
I.Soletta, PROGETTO IRDIS

A more detailed analysis of the cooling curve of a body is described in the module “Newton’s Law of Cooling”