DIRECTIONAL PROPERTIES OF LIGHT SOURCES

OBJECTIVE

To examine the spatial distribution of the radiation emitted by the real light sources.
The angular dependence of the radiant flux is explored and presented using polar coordinates.

CONTENTS

I. Theory.
   A. Basic concepts of Radiometry and Photometry.
   B. Determining the angular distribution of radiant intensity.
II. Apparatus for the experimental exploration.
III. Data acquisition.
IV. Data analysis (TI83).
V. Data analysis (MS Excel).

THEORY

A. Basic concepts of Radiometry and Photometry

The description and measurements of the propagation of the energy of the electromagnetic waves radiated by light sources uses twofold approaches.
The so called radiometric (physical) approach is based on the entire radiant power produced by the source while the photometric (physophysical) refers only to the part of the radiant power perceived by the human eye as light and human average response function. The spectrum of radiant energy waves that we call light is narrow, ranging from approximately 300nm to 750nm. Wavelengths shorter (UltraViolet) or longer (InfraRed) than these do not produce the visual response in the eye. Consequently we have two set of quantities and units used in measurements – see table 1. The symbols for radiometric quantities are analogous to the photometric counterpart.

All photometric quantities can be obtained from the their corresponding radiometric quantities on the basis of the spectral luminous efficacy human eye response
The relationship between the (physical) radiometric and the (psychophysical) photometric set units is established by the chosen value of spectral luminous efficacy for human vision: 683 lumens/watt.

In both approaches the basic concepts are similar and describe:

Power of the source

The light source is characterized by the total light output of a light source - total power radiated by it [in watts]. This is called Radiant power or Radiant flux Φ. In radiometry the unit of Φ is watt [W]. When referred to the sensitivity of the human eye and so called optical part of the radiated spectrum it determines the Luminous flux (or luminous power). The photometric unit of Φ is lumen [lm].

Intensity

The commonly used term intensity of light (“brightness”) refers to the energy radiated by the source into the unit solid angle in a unit time. This power per unit solid angle is called Radiant intensity I in watts/steradian. The photometric unit of the intensity (pointance, luminous intensity) is candela [cd]. The radiant intensity enables to describe spatial characteristics of the source. In general, it is given by:

\[ I(\Omega) = \frac{d\Phi}{d\Omega} \]
Experimental module: **Directional properties of light sources**

**Illumination**

In everyday life our visual reception is based on the effect produced by the light source at the surface of the surrounding objects. This means that we are interested in the amount of radiated energy, which reaches the observed surface element in a unit time. The **irradiance** \( E \) expresses radiation power (flux) received by the unit area of the illuminated surface. The unit of irradiance is **watt/m\(^2\)**. The respective photometric term is called **illuminance** and is given in **lux = lumen/m\(^2\)**.

The irradiance (as well as the illuminance) of a surface due to a point light source depends on the radiant intensity, the distance to the surface, and the orientation of the surface with respect to the source:

\[
E = \frac{\text{radiant flux} \: \Phi}{\text{area} \: A} = \frac{I \cdot \Omega}{A} = \frac{I \cdot A}{A \cdot r^2} = \frac{I}{r^2}
\]

where flux in a defined solid angle is:

\[
\Phi = I \cdot \Omega
\]

and solid angle:

\[
\Omega = \frac{A}{r^2}
\]

![Fig.1. Radiant flux \( \Phi \) and radiant intensity \( I \)](image)

**Solid Angle:**

The solid angle \( \Omega \) refers to the cone cut out from the sphere. Solid angle is related to the area \( A \) intercepted by the cone on the surface of a sphere of radius \( r \) centered on the cone vertex: \( A/r^2 \)

The unit of the solid angle is **steradian [sr]**.

![Fig.2. Solid angle](image)

For the surfaces not perpendicular to the direction of light propagation the illumination expression \( E \) should be modified in order to deal with the effective (projected) area of the illuminated surface – see the fig.(1):

\[
E = \frac{I}{r^2 \cos \phi}
\]

where \( \phi \) is the angle between the normal to the surface and direction of the propagation of the light energy.
Experimental module: **Directional properties of light sources**

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## The inverse square law.

The illumination produced by the point light source, which radiates equally in all directions follows the so-called Inverse square law.

It expresses the fact that amount of energy that passes through the unit area drops with the distance from the source. The total power irradiated by of the source (radiant flux) in all directions (into the full solid angle) remains constant while the total area of the sphere increases with the square of the radius.

$$ E \equiv \frac{I}{r^2} \quad (5) $$

So, the power per unit area drops – compare fig. 1. The general character of the inverse square law applies to many other phenomena based on point sources e.g. gravity pf point masses, electrostatic field of point charges etc.

### Table 1. Radiometric and photometric quantities.

<table>
<thead>
<tr>
<th>Radiometry</th>
<th>Photometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
<td><strong>symbol</strong></td>
</tr>
<tr>
<td>Radiant Energy</td>
<td>$Q$</td>
</tr>
<tr>
<td>Radiant Flux</td>
<td>$\Phi$</td>
</tr>
<tr>
<td>Irradiance</td>
<td>$E$</td>
</tr>
<tr>
<td>Radiant Intensity</td>
<td>$I$</td>
</tr>
<tr>
<td>Radiance</td>
<td>$L$</td>
</tr>
</tbody>
</table>

### B. Determining the angular distribution of radiant intensity

An ideal point source spreads its radiation equally in all directions without a limit to its range and obeys the inverse square law.

Precise knowledge of the angle-dependent distribution of radiation is necessary for many applications of the light sources from household lamps to car headlights.
The objective of directional measurements is to establish magnitude of radiant flux at certain angle at a certain distance from the source. This requires use of particular graphic representation of data.

Polar coordinates are used for directionality plots such that the magnitude of the response of the light detector corresponds to the distance from the center point at any angle. The Polar Coordinate system is especially useful in situations where information is most conveniently expressed in terms of distance from the origin.

The single point A in a polar coordinate system has two coordinates (see fig. 4): Radial coordinate \( r \) and the angular coordinate \( \theta \). Radial coordinate of a point A represents its distance from the origin \( r=OA>0 \) while the angular coordinate \( \theta \) is the angle measured counterclockwise from the polar axis (the positive x axis in respective cartesian system) to the segment OA connecting the point to the origin. By convention, we take the angle range to be \(-\pi < \theta < \pi\).

Converting between cartesian and polar coordinates is easy:

\[
\begin{align*}
x &= r \cos \theta \\
y &= r \sin \theta
\end{align*}
\]

Fig. 4. Polar coordinates

For real light sources the emitted radiant flux has non-uniform spatial distribution (fig. 5).

Fig. 5.

An example of angular distribution of the radiation emitted by a household incandescent bulb.
The radiometric measurements can be performed using the device that allows one to change the distance and the relative angle at which detector views the source (goniometer) – see fig. 6. The light detector measures the irradiance, which can be easily related to the radiant intensity.

Observation of the angular distribution can be accomplished by either moving the detector around a stationary source or by tipping and tilting the source while the detector remains in a fixed position (fixed distance \( r \)).

The angle \( \theta \) is adjusted by rotating the source about its mechanical axis or about an axis perpendicular to the mechanical axis. The detector is used to determine irradiance \( E \) arising as a result of the flux incident on the detector as a function of the rotation angle.

As the distance \( r \) and as well as the orientation of the detector’s active area \( \varphi \) remains unchanged for all measurements the recorded irradiance \( E \) vs. angle dependence expresses the radiant intensity \( I \) vs. angle dependence (compare equation 4.).

**APPARATUS FOR THE EXPERIMENTAL EXPLORATION.**

Determining the angular distribution of radiant intensity can be performed using the simple goniometer consisting of standard lamp handle placed on the rotating stage and fixed bench with adjustable light detector holder see fig. 7.

The light sensor uses a phototransistor with an active area positioned at a fixed distance from a light source. The detector measures relative irradiance \( E \). The units of irradiance are milliwatts per square centimeter. The light sensor’s output is a voltage that is linearly proportional to the amount of irradiance it senses.

This configuration assumes a point source, therefore the inverse square law holds true and the irradiance readings can be referred to the radiant intensity \( I \).
Experimental module: **Directional properties of light sources**

Experiment uses: (see Fig.7):

i. goniometer  
i. lamp housing E14  
ni. incandescent lamp(e.g. 15 - 40W)  
iv. Calculator Based Laboratory unit CBL  
   http://www.vernier.com/legacy/cbl/index.html  
   or CBL2  
v. Light probe (standard CBL or Vernier’s LS BTA)  
vi. Graphic calculator TI83, TI83 Plus, TI 83 Plus SE.  
vii. unit-to unit cable (standard).  
viii. Program: PHOT1, available for download at:  
ix. TI-GRAPH LINK™ (optional) cable  
   #serialwin  
   and software  
x. Personal computer with TI Connect™ software (optional)  
   Description:  
   Download:  

Practical notes about the apparatus setup.

- The Light probe should be connected to the channel CH1 of the CBL unit.  
- The detector housing should be fixed at the distance approx. 1m from the source with active area directed towards the source.  
- One should adjust the height of the detector’s mount with the center of the light source.  
- The ambient light should be reduced and kept stable during the experimental session.

**DATA ACQUISITION (TI 83)**

In the experiment the light sensor measures relative irradiance $E$ in milliwatts per square centimeter as a function of the angle $\theta$ between the present position of the axis of the source and the reference direction. The angle is controlled manually and can be chosen from $-180$ to $180$ degrees. The 0 value corresponds to the direct exposure of the detector on the source.  
Data can be presented in the rectangular coordinates as the $E(\theta)$ plot or in the polar coordinates where polar coordinate represents angle and radial coordinate represents the irradiance $E$.

The experiment is controlled by the calculator by means of the preloaded calculator program PHOT1.

Experimental procedure is divided into the preparatory part and the data acquisition.
**Preparation:**

Make all connections as shown on the Fig.7.

Turn on the calculator and the CBL.

Measuring the ambient light should precede the main measurements.

1. Launch the program PHOT1 by choosing its name from the **NPEK** menu.
2. Choose ZERO PROBE from the program menu – Fig.8.
3. Keep the light source off and expose light sensor to the background. Start collecting data – Fig.9.

   ![Fig.8.](image)

   ![Fig.9.](image)

The collected irradiance value will be subtracted from any other collected irradiance data.

The described calibration is to be done once during the experimental session provided that the background illumination does not change.

**Data collection**

1. Adjust the source stage at the angle 0 with respect to the reference mark (line connecting source and detector). Check position of the light detector. Its axis should exactly match the source.
2. Choose option 1: COLLECT DATA from the main menu.
3. Turn on the light source.
4. Start collecting data when ready. – Fig.10.
5. Introduce the chosen angle value when prompted (Fig.11). Note: angle values should be from 0-180 deg and 0-(-180) deg. Negative values represent the right hand turn from zero.
6. Take another data point for different angle by choosing 1: MORE DATA option from the DATA COLLECTION menu – Fig.12.
7. Finish collecting data by choosing 2: STOP AND GRAPH.
8. Choose the plot mode from the DATA PRESENTATION menu – Fig.13.
9. The RECT COORD prepares the $E(\theta)$ plot in rectangular coordinates – Fig. 14.
10. The POLAR COORD option transforms data into the polar coordinates and makes polar plot. - Fig.15. 
11. If you are not satisfied with the obtained data, repeat using the same or new settings (new calibration is not necessary). Choosing 1:COLLECT DATA from the main menu – Fig.8. 
12. Stop measurements choosing 2: QUIT from main menu – Fig.8. 
13. Collected data are stored in a calculator's memory and you can proceed using the standard calculator's functions. Now you can disconnect the CBL from the calculator.

DATA ANALYSIS (TI83)

Further analysis can be performed using tools implemented in calculators (or other analytical software tools such as MS Excel spreadsheet).

Collected data are stored in the calculator's lists:
- angle in degrees - List L1
- irradiance $E$ in watts/cm$^2$ - List L2

Transformed data for polar plot:
- $x$ component of the irradiance calculated from: $x = E \cdot \cos \theta$ in the list L3
- $y$ component of the irradiance calculated from: $y = E \cdot \sin \theta$ in the list L4

Exemplary data are available for a download in the following calculator type files:
- degree : as the file - Data sample/TI83/ L1
- irradiance - Data sample/TI83/ L2
- $x$ component of the irradiance - Data sample/TI83/ L3
- $y$ component of the irradiance - Data sample/TI83/ L4

Experimental data plot in the rectangular coordinates is defined as Plot1 (L2 vs. L1) while the polar plot is defined as Plot2 (L4 vs. L3) and can be recalled by STAT PLOT menu.

Analysis of plots

Analysis of the obtained angular distribution of radiant intensity can be done qualitatively by exploration of the plots. The detector gives the irradiance values however, as all other parameters are fixed (distance, orientation and active area of the sensor) the recorded values can be treated as representing the radiant intensity. The standard rectangular coordinates enable to easily compare distribution irradiance (intensity) magnitudes. The coordinates of data points retrieved by [TRACE] option give opportunity to directly examine values at given angle – Fig.16.

In engineering practice the angular distribution of radiant intensity is presented with use of the polar coordinates. The module (radial coordinate) presents magnitude of irradiance (intensity) while the polar coordinate the angle of observation.
Experimental module: Directional properties of light sources

The original recorded data correspond to the rectangular type of plot. So, they should be transformed in order to present them in polar system. The TI83 calculator does not offer polar plots of data stored in lists. So, one should transform the collected data in such a way that distance of the data point from the origin in the XY system represents the magnitude of irradiance (radial coordinate) while the angle between directional segment and 0X axis gives the polar coordinate – compare fig.4. The transformation equations are:

\[ x = E \cdot \cos \theta \quad y = E \cdot \sin \theta \] (6)

These new coordinates are calculated within the PHOT1 program and stored in the lists L3 and L4 respectively.

The polar plot is defined as PLOT2 (L4, vs. L3) and can be recalled from the STAT PLOT menu.

Exploration of this plot in terms of polar coordinates calls for change of the plot format (PolarGC) from the FORMAT menu – Fig. 17.

Then on the plot screen one can explore polar coordinates using cursor keys – Fig. 18.

Polar plot clearly shows spatial distribution of the radiant intensity of the source. The examined incandescent lamp radiates non uniformly (Fig. 18). Majority of the energy is radiated within the 100-degree angle (from −50° to +50°) while very limited energy is radiated backwards (180°).

DATA ANALYSIS (MS EXCEL)

Analysis can be performed using tools implemented in calculators and personal computer functions offered by the MS Excel software.

Transfer of experimental data to the personal computer.

After completing the experiment data could be transferred from the graphic calculator to the personal computer.

TI GRAPH LINKTM cable supported by the TI ConnectTM software offer tools enabling exploration of the contents of calculator (TI DEVICE EXPLORER) and data edition (TI DATA EDITOR).

Data collected in the experiment are stored in the calculator’s lists:
- angle in degrees - List L1
- irradiance mW/cm² - List L2

Within the TI Connect™ program - the TI DEVICE EXPLORER one can save calculator’s lists on the computer’s hard disk and then open them within TI DATA EDITOR. Option Special Lists Export provides opportunity to save the chosen list as the Excel Comma Separated type file (*CSV file). Such a file could be open and explore within the MS ExcelTM spreadsheet software.
Experimental module: **Directional properties of light sources**

Exemplary data are available for a download in the following files:
- angle: as file - Data sample/MSExcel/angle
- irradiance: as file - Data sample/MSExcel/intensity

**Analysis of plots**
Create a spreadsheet and import the data from the files angle and intensity. Make the plot $E(\theta)$ from the raw experimental data. Choose scattered type of the plot – Fig. 19.

![Fig. 19. Irradiance vs angle.](image)

MSEExcel does not offer the creation of the plots in the polar coordinates. There are two solutions of this problem. One uses the rectangular coordinate plot from transformed data. So, one should transform the collected data in such a way that distance of the data point from the origin in the XY system represents the magnitude of irradiance (radial coordinate) while the angle between directional segment and 0X axis gives the polar coordinate.

The transformation equations are:

\[
\begin{align*}
x &= E \cdot \cos \theta \\
y &= E \cdot \sin \theta
\end{align*}
\] (6)

However the MS Excel trigonometric functions call for the angle values in radians instead of degrees. So, the angle values should be transformed into radians before transforming irradiance data.

Use transformation:

\[
\alpha[rad] = \alpha[deg] \cdot \frac{\pi}{180}
\] (7)

Transformed data can be used for preparation of the semi polar plot- Fig. 20.

![Fig. 20. Angular distribution - semi polar plot.](image)
There is another option for creation of the polar plot.

One can use freeware Polar Plot add-in for the MS Excel developed by Andy Pope. This convenient tool is freely available at [http://www.andypope.info/charts/polarplot.htm](http://www.andypope.info/charts/polarplot.htm). With the help of the Polar Plot one can create the polar plot directly from the raw experimental data – Fig. 21.

![Fig. 21. Polar plot created using the Polar Plot add-in.](image)

Note:

The complete numerical analysis is presented in the MSEexcel file:

Data sample/MSEexcel /lightang